PRE-SERVICE TEACHERS' AND STUDENTS' (MIS)CONCEPTIONS ABOUT THE EQUAL SIGN

A Thesis

by

KATHERINE NICOLE VELA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2011

Major Subject: Curriculum and Instruction

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ABSTRACT

Pre-Service Teachers' and Students' (Mis)Conceptions

About the Equal Sign. (December 2011)

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The objective of this thesis was to investigate pre-service teachers and student misconceptions of the equal sign, and then offer suggestions to pre-service teachers, teachers, university programs, and schools to prevent common misconceptions from occurring in classrooms. Some students do not realize the equal sign can have two different functions, operational and relational. There are several different reasons for this misconception, beginning with the lack of defining what the equal sign is and what it means in the classroom.

In the first study, eighteen participants were interviewed to explain their responses when evaluating student work to gain an in-depth knowledge of pre-service teachers' perceptions of the equal sign and their ability to evaluate a students' response to a specific math task. Results showed that pre-service teachers have a better understanding of the equal sign and may be ready to teach the equal sign as a relationship between numbers. Furthermore, pre-service teachers would benefit greatly from evaluating students' work and looking for common misperceptions that students may have.

In the second study, six fifth grade classes were studied to determine if there was a positive relationship for teaching atypical type equivalence statements to students and performing better on equivalence questions. Three classes from Spring 2011, were administered a test; two of the test items were used to analyze their understanding of the equal sign. In Fall 2011, another three fifth grade classes participated in lessons, which required students to analyze atypical type equivalence statements, and then they were given the same two test items. Results from this study supported the use of atypical type equivalence statements because more students in the experimental group correctly responded to the two items and were also able to justify their responses with work that exemplified good understanding of the equal sign as being a relationship.

Both of these studies support increasing student and pre-service teachers understanding of the equal sign and the misconceptions students have regarding the equal sign. University programs and schools should utilize these results to require pre-service teachers and teachers to evaluate student work to identify common misconceptions and teach the equal sign as a relationship between both sides and not as an operation.

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CHAPTER I

INTRODUCTION: THE IMPORTANCE OF RESEARCH

The equal sign is indispensable in mathematical understanding, from basic arithmetic to more advanced mathematics; students need to have a solid foundation of this concept in order to be successful (McNeil & Alibali, 2005). Despite how important this concept is, very little class time is spent discovering the idea of the equal sign and equivalence; thus, students develop their own conceptions about what the equal sign means based on their limited experiences, and carry these misconceptions with them throughout different grade levels (McNeil & Alibali, 2005; Alibali, Knuth, Hattikudur, McNeil, & Stepehens, 2007). Furthermore, students end up developing an operational sense of the equal sign because of instructional activities and available materials (McNeil, 2007; McNeil, 2008; McNeil et al., 2006; Prediger, 2009), and never discover the relationship between the left and right sides of the equal sign (Falkner, Levi, & Carpenter, 1999; McNeil, 2008).

Purpose

The purpose of conducting the study connected with this thesis is to increase knowledge of pre-service teachers' abilities to scrutinize students' misconceptions and their skills in providing instruction to decrease these misconceptions. Both of my articles jointly, will present information for university pre-service teacher educators and

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This thesis follows the style of *Journal of Mathematics Teacher Education.*

current teachers for increasing student understanding of equivalence and the equal sign. First, I will examine pre-service teachers' capabilities for identifying misconceptions and correcting these misconceptions. Secondly, I will explore student responses to equivalence problems and offer suggestions for increasing student understanding.

Research Questions

The purpose of this study will be to gain knowledge and understanding of pre-service teachers and students' misconceptions of the equal sign, and provide suggestions to deter these misconceptions from occurring.

The following research questions directed me throughout this process in gathering insights into pre-service teachers' and students' perceptions about equivalence.

1. How well do pre-service teachers' identify misconceptions in students' work? How do pre-service teachers correct these misconceptions? How can pre-service teachers teach equivalence?
2. Will students respond correctly to equivalence problems? How do students prove/justify their solutions when working with problems involving equivalence?

CHAPTER II

PRE-SERVICE TEACHERS' PERCEPTIONS OF STUDENT MISCONCEPTIONS OF THE EQUAL SIGN

Many students have common misconceptions about the equal sign and what the equal sign actually means. There are several different reasons for these misconceptions beginning with the lack of defining what the equal sign is and what it means in the classroom. Student misconceptions concerning the equal sign could be the result of minimal teacher background knowledge and/or available textbooks and materials (Ball, 1990; Capraro, Ding, Matteson, Capraro, & Li, 2007; Davis & Simmt, 2006; Falkner, Levi & Carpenter, 1999; Jones & Pratt, 2005; Knuth, Stephens, McNeil, & Alibali, 2006; Li, Ding, Capraro & Capraro, 2008; Prediger, 2009; Sherman & Bisanz, 2009).

Literature Review

Different Meanings for Equality

Many researchers highlight the difference between the two meanings for equality, operational and relational (Filloy, Rojano, & Solares, 2003; Knuth et al., 2006; Prediger, 2009). Despite the fact there are two different definitions, only the operational meaning has been stressed in many classrooms (McNeil, et al, 2006; McNeil, 2007; McNeil, 2008; Prediger, 2009). This operational meaning involves doing something, computing the sum, or executing all given operations. For instance, 5+7=12, the students add five and seven together which results in twelve. In other words, an operation will equal the answer, or the answer follows the equal sign. Many students view the equal sign as a broadcast that the answer will be next, and not that both sides are in fact equal (Knuth et al., 2006). This misconception has been reinforced with the use of the four function or scientific calculators in the classroom; first students enter the problem 5+7; then press the equal sign and see the answer (Jones & Pratt, 2005; Li et al., 2008). For most students, this was their only understanding of the equal sign, so when students began algebra or solved algebra type problems they had a difficult time (Herscovics & Linchevski, 1994; Sherman & Bisanz, 2009).

Relational was the second meaning of equality when students understand the equal sign to be a balance (Alibali, 1999; Li et al., 2008), where both sides of the equal sign represent the same value. Students who do not view the equal sign as relational are less successful in algebra type questions than students who understand the equal sign as a relationship between both sides (Knuth et al., 2006). Students have to see the equal sign as a relationship in order to comprehend "complicated equations with operations on both sides of the equal sign (e.g., $5x + 7 = 2x - 11$)" (Stephens, 2006, p. 252). "Allowing children to focus on the relationships between quantities and sides of the equations, rather than on applying addition schemas" (Sherman & Bisanz, 2009, p. 98) promotes a deeper understanding of the equal sign and bridges the gap from elementary math to algebra. Knuth et al.'s study suggested very few middle school students see the equal sign as a relationship between both sides and later advocated understanding the equal sign may increase student achievement levels in algebra.

Reasons for Equal Sign Misconceptions

There could be several reasons for equal sign misconceptions beginning with the lack of defining what the equal sign is and what it means in the classroom. Student misconceptions could be the result of minimal teacher background knowledge as well as the textbooks and materials available (Ball, 1990; Capraro et al., 2007; Davis & Simmt, 2006; Falkner, Levi & Carpenter, 1999; Jones & Pratt, 2005; Knuth et al., 2006; Li et al., 2008; Prediger, 2009; Sherman & Bisanz, 2009). It is the role of the teacher to accurately define and provide students with various activities to be aware of the different meanings of equality, so students are engaged in problem solving which develop a deeper understanding of the equal sign. Furthermore, "teacher preparation materials in the United States, student texts, and professional development should be aligned to focus-on equality as a statement of relation" (Li et al., 2008, p. 210). Providing teachers with a more in depth study of the concept of the equal sign will in turn provide students with a better understanding of what the equal sign means, which will then positively affect student performance in advanced mathematics.

Teacher background knowledge

Teachers play a vital role to the children in their classroom. This vital role involves teachers being conscious of where their students' understanding began, appropriately teaching content in meaningful ways, thus adapting instruction to their students' needs, and being aware of the direction their students are headed. The mathematics understandings most teachers bring with them are "rule-bound and thin" (Ball, 1990, p. 449). Teachers must have an in-depth understanding of the mathematics they expect their students to acquire, in order to determine misconceptions when reviewing student work (Blanton & Kaput, 2003; Davis & Simmt, 2006; Stephens, 2006). Teachers must be familiar with their students' strengths and shortcomings in order to teach mathematics so students understand.

Before teaching any lesson, teachers need to understand their students' prior knowledge and recognize where their students' understandings began. Most teachers lack sufficient background knowledge of algebra because most have not been in an algebra class since high school (Blanton & Kaput, 2003). Furthermore, pre-service teachers are not familiar with the equal sign misconception (Herscovics & Linchevski, 1994), so "it is not surprising that they would not value problem-solving strategies that enhance this understanding over more familiar, computationally-based strategies" (Stephens, 2006, p. 274). First, teachers need to be aware of this common misconception, and then realize the misconception is difficult for students to correct. If the misunderstanding is not addressed, it will lead to confusion in more advanced mathematics, such as algebra (Stephens, 2006).

Additionally, teachers need to be able to correctly identify student misconceptions (i.e. student thinking and learning processes) and decide on a plan of action to help students end this misconception (Herscovics & Linchevski, 1994). This was known as adaptive teaching, which required teachers to adapt their teaching to student's individual needs (Prediger, 2009). Adaptive teaching will motivate teachers to plan how to appropriately teach the material in meaningful ways that are specific to their own students, thus increasing students' understanding of the equal sign (Alibali, 1999; Prediger, 2009; Stephens, 2006).

Learning mathematics conceptually rather than learning a set of rules or actions will encourage students to become successful problem solvers (Rittle-Johnson & Alibali, 1999; Stylianides & Ball, 2008). One study conducted by Pirie & Martin (1997), reviewed a conceptual approach to teaching linear equations, and showed growth in mathematical understanding of the students after the teacher taught mathematics by instructing students to think about the equation rather than simply learning a set of rules or actions. Students were encouraged to consider what would satisfy the equation, rather than "What do I do" (Pirie & Martin, 1997, p. 178). Students were also expected to discuss their mistakes and the reason for these mistakes. By collaborating and utilizing think-alouds in the classroom the students were deterred from making similar mistakes. This example demonstrated a teacher who understood that "conceptual knowledge may have a great influence on procedural knowledge than the reverse" (Rittle-Johnson & Alibali, 1999, p. 175).

While finding the best ways to appropriately teach content to students, teachers also need to understand the direction in which their students are headed. For instance, Knuth et al.'s (2006) suggested, "that understanding the equal sign is a pivotal aspect of success in solving algebraic equations" (p. 309). In other words if teachers were only teaching an operational meaning of the equal sign, students will be less successful in future mathematics (Knuth et al.; McNeil, 2007). It was imperative that teacher's help students realize the equal sign should be viewed not an operation but a relationship between both sides. Teachers accomplished this by engaging students in problem solving that required students to value the relationship between both sides of the equal sign.

However, in order to be able to teach students effectively and increase student achievement, teachers must have a solid understanding of the mathematics to recognize the importance of higher level thinking tasks and be conscious of research concerning student misconceptions (Stephens, 2006). In order to hone in on student misconceptions, pre-service teachers must be asked to evaluate student work in methods courses in order to gain insights into how students think and learn (Stephens, 2006). Evaluating student work with misconceptions and identifying strategies to deter these misconceptions in methods courses will increase teacher background knowledge so they are equipped to teach their students in a variety of ways instead of just learning mathematics as a procedure (Alibali, 1999; Prediger, 2009; Stephens, 2006).

Current textbooks and materials

Another reason for the lack of knowledge about the equal sign in classrooms is due to current textbooks and materials. One study compared U.S. textbooks to Chinese textbooks and found "textbooks do little to mitigate the problem in the United States while in China students are able to interpret the equal sign as a relational symbol of equivalence" (Capraro et al., 2007; Li et al., 2008). Reiterating the notion that students predominantly use the operational meaning of the equal sign and rarely see the equal sign as a relational symbol in textbooks (Capraro et al., 2007; Li et al., 2008; McNeil et al., 2006). Chinese students were instantly introduced to the concept of the equal sign as a relation between opposite sides; these differences in the textbooks and methodology could explain why Chinese students outperformed students in the U.S. on mathematics assessments (Capraro et al., 2007). A further reason for the misconception could be due to the lack of textual presentations in the textbooks (Li et al., 2008; McNeil et al., 2006).

Textbooks in the U.S. do not provide activities or lesson examples of how to teach the equal sign; therefore, most teachers rely on their experiences and understanding of the equal sign (Li et al., 2008). If teachers do not have a solid background or understanding of the equal sign themselves, they will not be able to effectively teach their students the different meanings of the equal sign with the resources they are provided (Ball, 1990; Blanton & Kaput, 2003).

Methods

The purpose of this study was to investigate pre-service teachers' perceptions of student work and the relational use of the equal sign.

Participants

Eighteen pre-service teachers (2 males and 16 females) enrolled in an Integrated Mathematics and Science course at a large mid-south university were the participants. These pre-service teachers learned problem-solving strategies and had just completed their final exam. Participants were asked to explain their responses for three different test items. A structured interview was used to acquire a more in-depth explanation of the participant's response to one test item.

Materials

Materials for this study included the final exam and a tape recorder. The exam consisted of 18 test items. Three of these test items were used for think-aloud interviews (Kuusela & Paul, 2000) and these interviews were recorded for analysis.

Procedures

Upon completion of the final exam, participants participated in think-aloud interviews by three interviewers to explore their responses on the three different test items. The interview process was a randomized block design to deter learning throughout the interviews (See Appendix A for example of interview processes). One test item was explored in greater detail to analyze pre-service teachers' knowledge when interpreting students' work and understanding of the equal sign (See Appendix B for the test item). Pre-Service teachers were asked to first explain the student's process and then evaluate the student's process and answer. Participants had to decide whether the student's answer was correct or incorrect, showed knowledge of arithmetic was incorrect but the answer was correct, or did not match the problem. Once choosing their response, the pre-service teachers had to defend their answer with arguments during the interview. Afterwards, all interviews were transcribed and responses were analyzed looking for common themes.

Analysis

Grounded theory and constant comparisons (Corbin & Strauss, 2008) were incorporated to determine common themes within the think-aloud interviews. After reviewing the interviews, the data from the interviews were coded into these different themes and several themes were evident. Themes included pre-service teachers' responses to the test item (See Appendix B), evaluating the student' process, what the student could have done to improve their work or process, the concept of the equal sign, and how the pre-service teachers would teach this problem in their own classroom. A pie chart will be used to display the different responses to the test item, in order to determine which response was predominantly chosen.

Results

The research questions for this study were aligned with the questions embedded in the test item that participants responded to. Therefore, the results were reported following this same sequence.

Pre-Service Teachers' Responses

Of the 18 pre-service teachers interviewed, 4 chose A, the student's answer was correct, 1 chose both B and C, which stated the students answer was incorrect and the students' work showed knowledge of arithmetic, and 13 chose letter D, the work was incorrect, but the answer was correct, and none chose the students' work does not match the problem. As demonstrated in Figure 1, of the pre-service teachers interviewed, 90% of the participants believed the students answer was correct, those who chose A and D, but 69% believed the student answered the question accurately, but did not show their work properly, those participants who chose D. Several of these pre-service teachers evaluated the students' process and explained how the student could have shown their work more accurately to receive full credit.

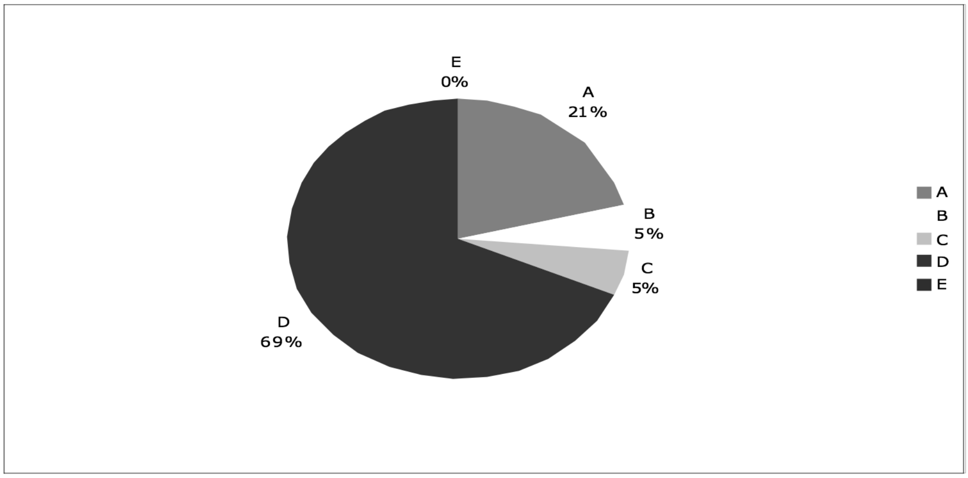

Figure 1. Pre-Service teachers' response to test item.

Evaluating the Student Process

Each pre-service teacher agreed the students' work illustrated they had read the problem and performed the actions described in the problem. Each participant was also in agreement that the student understood when to add or subtract the apples or oranges and that the student used a systematic process when solving the problem, by calculating every step right after the previous step. Also, the pre-service teachers decided that the arithmetic was correct, but the process was incorrect. One participant acknowledged the student "kind of like jumbled the whole equation together. Pretty much looked at it step-by-step and just used equal signs." Another participant remarked "if you look at it, it says 4+3=9, or 4+3=7–1, which equals 6–2, which equals 4+2, which equals 6+3, which is not right." The participants then discussed how the student should have shown their work in order to get full credit.

What the Student Should Have Done

The think-aloud interviews revealed the participants' suggestions to make the students' work more accurate in order to receive full credit. There were several suggestions for showing work differently, but most agreed the student needed to find a better way to organize their work; either through a chart, breaking it into smaller steps, finding the total number of apples, the total number of oranges, and then adding both of these totals together to find the total number of fruit, or labeling their work as either apples, oranges, or total.

Only 5 of the 18 participants suggested splitting the different equations up, or using the equal sign correctly. For instance, one participant stated "the student should have done 4+3=7, and then taken down a new line and put 7–1=6." Along the same lines, another pre-service teacher suggested the student "break the equations apart." A different participant went further to advise students to "separate each arithmetic sentence, so that you can look at it and clearly see each step of the process so they can get their full credit." Separating each arithmetic sentence, leads to the concept of the equal sign, and how the student may not have a complete understanding of what the equal sign means.

The Concept of the Equal Sign

Of the 18 participants, only 10 discussed the students' misuse of the equal sign. Each of these 10 participants pointed out that the student had created equivalent statements that were untrue. For example, "if you look at it (the students work), it says 4+3=9, or 4+3=7–1, which equals 6–2, which equals 4+2, which equals 6+3, which…that's not right." One participant goes on to say "unfortunately they left the equal sign between 4+3 equals the 7–1, …and it's obvious given their math to get the correct answer, that they do know that 4+3 does not equal 7–1." Another pre-service teacher elaborated, "he (the student) only understood the function of the equal sign through an operational view. So 4+3=7, and then he continued on trying to compute, but 4+3 does not equal 7–1, which does not equal 6–2. And the student does not understand that the relationships that he was saying were true are (actually) not true." Since, the pre-service teachers noticed a weakness in the students' work; they subsequently explained how they would teach it in their classroom in order to increase understanding.

How They Would Teach It

The pre-service teachers explained in great detail how they would teach this same item in their classroom. Many of the participants mentioned more than one method they would employ to teach this item. Methods suggested included the use of manipulatives, solving the problem step-by-step, separating the problem by apples and oranges, labeling their work, and evaluating equivalence statements.

Manipulatives

Four of the pre-service teachers believed teaching with manipulatives and visuals would be the most appropriate way for students to show their work on this item. One participant believed this was best because then students "can really understand both, what addition and subtraction actually is, that you are taking it away." Another participant justified using manipulatives because using "pictures and showing them in a very visual way how you are adding and subtracting the fruits" would help make the problem easier to understand. Once students understand what is happening in the word problem, students can then work the word problem step-by-step in a linear fashion.

Step-by-step problem solving

Nine of the participants thought teaching the students to solve the problem step-by-step, instead of just one long equation, would be the most appropriate. The pre-service teachers acknowledged they could teach this item by computing each line of the problem step-by-step and writing each new step on a different line, creating equivalent relationships using the equal sign. A pre-service teacher commented that going down instead of across would be " more useful to students to see the process." Also, solving the problem step-by-step decreases the misuse of the equal sign. Another way to reduce the use of the equal sign could be to separate the problem into its different parts.

Separating parts

Seven of the participants thought that separating the apples and oranges was important when solving this problem. They believed separating the apples from the oranges would help students understand the problem and keep their work organized. One participant even remarked, "I remember in school they were like you can't add apples and oranges." Another participant believed it was important to separate the apples from the oranges just in case the teacher asked another question concerning this same problem. She stated, "The students would have to do it all over again because they didn't separate it." When separating the problem into its different parts, it is very important for the student to label each part so they do not get confused.

Labeling

Three of the participants were also very adamant they would teach their students to label their work in order to help them remember what each of the numbers represent. One pre-service teacher explained why this was so important, "I would advise the student instead, that they need to label apples and oranges and fruit, so that it is clear, and clearly communicated how they worked the problem." Labeling is also important for the students, because it helps them remember what each number is representing, like one participant stated, "make sure that you are either taking it as a total fruit or as total number of apples or total number of oranges." Without labeling, students may not be able to look back at their work and remember what each number represented.

Evaluating equivalent statements

Finally, two of the pre-service teachers mentioned having students evaluate equivalent statements in order to reinforce the concept of the equal sign. One of them stated they would "have a lesson on the different functions of the equal sign, so while the students see the equal sign as doing things, I would rather try and show the students that the equal sign…can show relationships between numbers, rather than just telling them to do something." Having students evaluate equivalent statements reinforces the relational meaning of the equal sign.

Discussion

Many students and pre-service teachers have misperceptions of the equal sign; most of them have only encountered the operational definition of the equal sign. The item reviewed in this study (See Appendix B), is a prime example of how some pre-service teachers do not grasp the relational meaning of the equal sign. It provides evidence that most pre-service teachers view the equal sign as only an operation. There are several different reasons for this misconception beginning with the lack of defining what the equal sign is and what it means in the classroom.

Teachers should present a variety of problems involving the equal sign with different “arrangements of missing numbers, operators, or both” (Li et al., 2008, pp. 208-209). Teachers should challenge student misconceptions; as well as understand why being conscious of these misconceptions is important (Stephens, 2006). Students will continue to have these misconceptions and struggle with algebra type questions until teachers know and understand the importance of teaching the equal sign as more than an operational symbol, and students are provided with opportunities to see the equal sign as a relationship in equality tasks (Jones & Pratt, 2005; McNeil, 2007).

In this study, pre-service teachers were interviewed to gain a better understanding of how they would evaluate an equality task item completed by a student and how they would teach it in their classroom. Most of the participants (See Figure 1) (90%) believed the student answered the item correctly. However, 69% of the pre-service teachers did not believe the student *worked* the problem correctly. Most of the pre-service teachers realized working a problem this way, would create issues and difficulties for students in higher-level mathematics. Many suggested teaching the students how to break up each individual step, so that their work accurately represented what they were trying to convey.

One of the main reasons students have misconceptions about the equal sign is because teachers are employing only the operational definition in the classroom and not building a solid foundation for students to understand the equal sign as an equivalent relationship with the items on either side of the equal sign. Stephens (2006) agreed that teaching pre-service teachers how to evaluate students' work and look for misconceptions that would hinder learning in the future would have a positive impact on student understanding in mathematics, because pre-service teachers are being required to take an in-depth look at how students think and solve problems. Identifying students' misconceptions when they first begin will have a profound impact on their learning in the future.

Many of the pre-service teachers in this study were able to explain the students' processes and give advice for showing the work more accurately. This reinforces the fact these pre-service teachers were able to identify the misconception and offer suggestions to prevent the misconception from continuing. One of the pre-service teachers took the idea even further and suggested they would have a class lesson on evaluating equivalent statements to help students keep in mind that the equal sign means both sides should be equal. Another pre-service teacher suggested having a lesson on the different functions of the equal sign, so students would stop seeing the equal sign as doing something, but as a relationship between numbers.

Lessons evaluating equivalent statements are needed in the classroom, to help students realize the equal sign is not an operation but rather a relationship between both sides of the equation. More pre-service teachers should be required to view samples of student work and look for misconceptions in order to understand common misperceptions students may have. Reviewing student work and being aware of these misconceptions will assist pre-service teachers in preventing these misconceptions and focus them on looking for these situations in their own future classrooms.

The results indicate there is evidence that upon completion of a problem-solving course, pre-service teachers have a better understanding of the equal sign as compared to students in previous studies (Alibali, 1999; Prediger, 2009; Stephens, 2006) and may be ready to teach the equal sign as a relationship between numbers and not as an operation. However, pre-service teachers would benefit from evaluating students' work more often and looking for common misconceptions that students may have. This will require pre-service teachers rethink how they would teach mathematic concepts in their classroom.

The next step for this study would be to administer the test item and other tasks that require students to understand the relational meaning of the equal sign to a group of students and evaluate how the students solve these items and tasks. Permitting the researcher with better insights into how students approach these equal sign tasks and give them a chance to determine if a greater number of students understand the equal sign relationally, the researcher then would be able to offer suggestions and present tasks and items that will deter these misperceptions from occurring.

CHAPTER III

STUDENTS' MISCONCEPTIONS ABOUT EQUIVALENCE

Many elementary teachers do not realize the effects of only understanding the equal sign as an operation, but these effects are dire and will hinder students when they reach more advanced mathematics (McNeil & Alibali, 2005). For example, researchers asked students to solve 8+4=[] + 5, every student who was given this problem responded with 12 or 17 as their answer (Falkner et al., 1999). These students only have an operational understanding of the equal sign, they added 8+4 and got 12 as their answer, or added all three numbers, which resulted in 17 as their answer. This was because students were familiar with seeing equations with the operation on the left side and the answer to the right of the equal sign; they are not used to seeing operators on both sides.

Literature Review

Students who do not view the equal sign as relational are less successful in algebra type questions than students who understand the equal sign as a relationship between both sides (Knuth et al., 2006). "Allowing children to focus on the relationships between quantities and sides of the equations, rather than on applying addition schemas" (Sherman & Bisanz, 2009, p. 98) promoted a deeper understanding of the equal sign and bridged the gap from elementary math to algebra (Alibali et al., 2007; Stephens, 2006). Knuth et al.'s (2006) study suggested very few middle school students see the equal sign as a relationship between both sides and later advocated understanding the equal sign increased student achievement levels in algebra.

History of the Equal Sign

The equal sign has been studied for well over thirty years by researchers, all of which have noted some sort of misconception students have concerning the equal sign. Weaver (1971, 1972) noted students had a very hard time solving for missing numbers in a number sentence when the operation was on the right (as cited in Baroody & Ginsberg, 1983). For example, 14= ? + 4, these types of questions were hard for these students because students were not accustomed to seeing equations written with the operation on the right hand side. Next, in 1974, Collis believed that students were not mature enough to deal with equations until they reached the age of thirteen (as cited in Baroody & Ginsberg, 1983). Researchers continued to study the equal sign, and noted that students were inclined to focusing the equal sign as an operation, something you do, rather than relational or meaning the same on both sides (e.g., Behr, Erlwanger, & Nichols, 1976, 1980; Denmark, Barco, & Voran, 1976; Van de Walle, 1980 as cited in Baroody & Ginsberg, 1983). Denmark, Barco, and Voran (1976) came to a similar conclusion as Collis, who believed students were not intellectually ready to interpret the equal sign, and also found that classroom instruction added to students' lack of a relational understanding (as cited in Baroody & Ginsberg, 1983).

Byers & Herscovics (1977), extended prior research and stated if the equals sign was not viewed as a relationship between left and right sides, then once students entered algebra and began solving equations, the methods would not be meaningful to the student and the student would only memorize a procedure (as cited in Baroody & Ginsberg, 1983). In 1981, Keiran continued to advocate for researchers affirming students continued to have difficulties in mathematics because of their misconceptions about the equal sign (as cited in Alibali et al., 2007). Baroody and Ginsberg (1983) agreed with previous researchers who stated students continued to view the equal sign as an operator, but disagreed with researchers who believed students had these misconceptions because they were not mature enough to understand the equal sign. Baroody and Ginsberg also concluded that appropriate instruction could change students' views of the equal sign. Recent researchers still believe students are holding strong to the operational understanding of the equal sign, but data implies once student are able to understand the equal sign relationally, they will be able to distinguish between the equivalence of two equations (Alibali et al., 2007).

Improving Student Understanding

Many materials used in mathematics classrooms maintain an operational view of the equal sign, where the operations are on the left side of the equal sign and the blank is on the right side, despite researchers advocating for a relational understanding (Alibali et al., 2007; McNeil et al., 2006; McNeil, 2008). These typical operational equations only reinforce to students the equal sign means the answer goes next or find the sum, difference, product, or quotient (Alibali et al., 2007; McNeil et al., 2006; McNeil, 2008). Students then create their own generalizations about the equal sign, and the context they are presented with daily effect these generalizations or misconceptions (McNeil & Alibali, 2005). Students should be taught about equivalence through the use of atypical arithmetic problems, from the beginning, in order to gain a deeper understanding of the equal sign (Baroody & Ginsberg, 1983; McNeil, 2008).

In order to improve student understanding of equivalence, McNeil (2008) suggested using atypical equations to break the cycle and allow students' to view both sides of the equal sign as interchangeable elements. For example, McNeil recommended teachers present students with equations with the operators on the right hand side (15=5x3), operators on both sides of the equals sign (5x3=10+5), or simply the same number equal to itself (15=15). McNeil believed that by providing students with various examples concerning the equal sign they will gain a clear, concise relational view of the equal sign, where both sides are equal or the same and not the equal sign means the student should do something.

Even though research has suggested that many elementary students only view the equal sign as an operator, later students learn to view the equal sign as a relational symbol (McNeil & Alibali, 2005), most likely occurring because of their continued experience with the equal sign. Furthermore, research suggested if teachers learn to acknowledge students misconceptions and use results from research to support their instruction, their students will gain a deeper understanding of mathematical concepts (Stephens, 2006).

Impact on Student Learning

Students who develop a relational understanding early on have greater success on algebra type questions and outperform students who only have an operational understanding of the equal sign (Alibali et al., 2007; Knuth et al., 2006). Since, individuals understand the concept longer before they can employ these conceptions, it is crucial that students begin analyzing the equal sign as relational as early as possible

(Alibali et al., 2007; McNeil & Alibali, 2005). Students will struggle at first, but ongoing various activities regarding the equal sign will help students develop a deeper understanding of the equal sign as a relationship between both sides (Alibali et al., 2007; McNeil & Alibali, 2005). Engaging students in classroom discussions and presenting them with atypical equations will help students see the equal sign as more than an operation, but as a relationship between the left and right sides of the equation (Falkner et al., 1999; McNeil, 2008). This will lay the foundation for more advanced mathematics which require a relational understanding of the equal sign (Falkner et al., 1999)

Methods

The purpose of this study was to investigate student perceptions of the equal sign and how these perceptions guide their thinking and solving of equivalent statements, and to see if giving students experience solving atypical equivalence statements will have a positive impact on solving equivalent type questions.

Participants

Six classes, 137 fifth grade students total ($n_{\text{control}} = 72$, $n_{\text{experimental}} = 65$), all having the same teacher were participants in this study. The students were from a Title I School located in a mid-south Texas community. The demographics of the elementary school include 19.8% Black, 57.6% Hispanic, 22.4% Caucasian, and 0.2% Asian or Pacific Islander (See Figure 2). The control group consisted of the entire 5th grade monolingual classes for the 2010-2011 school year, and the experimental group consisted of the entire 5th grade monolingual classes for the 2011-2012 school year. The two groups were comparable with their mathematics skill level. Three fifth grade classes (Spring 2011), control group (*n*=72), had recently finished a unit on Patterns, Relationships, Probability, and Algebraic Reasoning and were taking a district-mandated test covering these topics. These students were not introduced to the equal sign using a variety of examples concerning the equal sign like McNeil (2008) suggested. The other three fifth grade classes (Fall 2011), experimental group (*n*=65), were presented with a variety of examples dealing with the equal sign, for example, operators on the right hand side, operators on both sides of the equals sign, and the same number equal to itself, like one researcher (McNeil, 2008) suggested. These students were then given two of the same questions from the Patterns, Relationships, Probability, and Algebraic Reasoning, as a comparison with the control group.

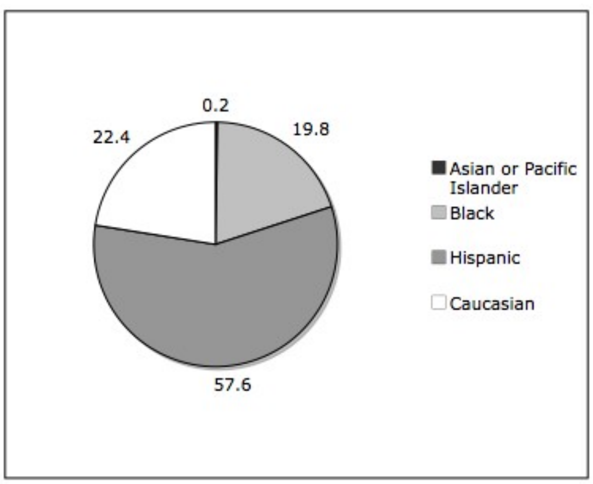

Figure 2. Percentages of elementary school student demographics.

Instrumentation

Materials for this study included the district-mandated Patterns, Relationships, Probability, and Algebraic Reasoning Test, for the control group consisting of 15 multiple choice test questions. Two of the test questions concerning equivalence were given to the experimental group after they were taught using various types of equivalence statements. These two equivalence questions were examined in further detail.

Procedures

Procedures different slightly between groups.

Control group

Upon completing a three-week unit (Spring 2011) dealing with Patterns, Relationships, Probability, and Algebraic Reasoning students were given a cumulative district-mandated test covering these topics. The test included 15 multiple-choice questions, and students' scores were based on the accuracy of each question. Two questions were examined in more detail to gain a deeper understanding of utilizing atypical equivalence equations (See Appendix C for the two test questions). Throughout the unit students had not encountered atypical equivalence equations. By analyzing the results of these two questions the researcher could draw conclusions about the students' conceptions concerning equivalence and the equal sign.

Experimental group

Students participated during the Fall of 2011 in a series of lessons where they evaluated atypical equivalence statements in order to build their content knowledge of the equal sign, and understand the equal sign as a relationship between the left and right hand sides of the equal sign and not only as an operation leading to the answer comes next (See Appendix D for samples of atypical equivalence statements used). After students showed mastery of these atypical equivalence statements, students were given the same two equivalence questions as the control group in order to compare results.

Analysis

The data were represented and modeled in several ways to help readers understand the data and results from the study. First, a spreadsheet was compiled containing student data of results from each equivalence question as well as how the student worked the question. Percentage correct from both the control group and the experimental group was also analyzed to draw conclusions about the effect of having experience evaluating atypical equivalence statements. If students in the experimental group have a higher percentage correct, the researcher can assume a positive effect of utilizing atypical equivalence statements in the classroom. Similarly, if students are able to prove their answer with a higher level of understanding of the equal sign, it will also support the use of atypical questions in early mathematics.

Grounded theory and constant comparisons (Corbin & Strauss, 2008) were incorporated to determine common themes within the students work. The data from the students work was coded into different themes and discussed for further review in the results section. A pie chart was utilized to display responses to each question, to determine if the correct response was predominantly chosen. This also allowed the researcher to draw conclusions about students' conceptions concerning the equal sign if they chose the wrong answer. A bar graph was also utilized to display common themes of student work, this allowed the researcher to examine how students show work when proving equivalence statements, and potentially offered suggestions about how future teachers and pre-service teachers should instruct students when solving equivalence questions.

Results

The purpose of this study was to determine if exposing students to atypical equivalence statements would have a positive effect on student work and responding correctly to equivalence questions. The groups were given the same two questions, but the experimental group was exposed to atypical type equivalence statements in order to stretch their understanding and meaning of the equal sign.

Control vs. Experimental Group Results

Sixty eight percent and 61% of the control group answered correctly question 1 and 2 respectively (See Figures 3 and 4, for control group results: question 1 and 2). Whereas, the experimental group who were previously exposed to atypical equivalence statements answered correctly question 1, and 2, 65% and 88% respectively (See Figures 5 and 6, for experimental results: question 1 and 2).

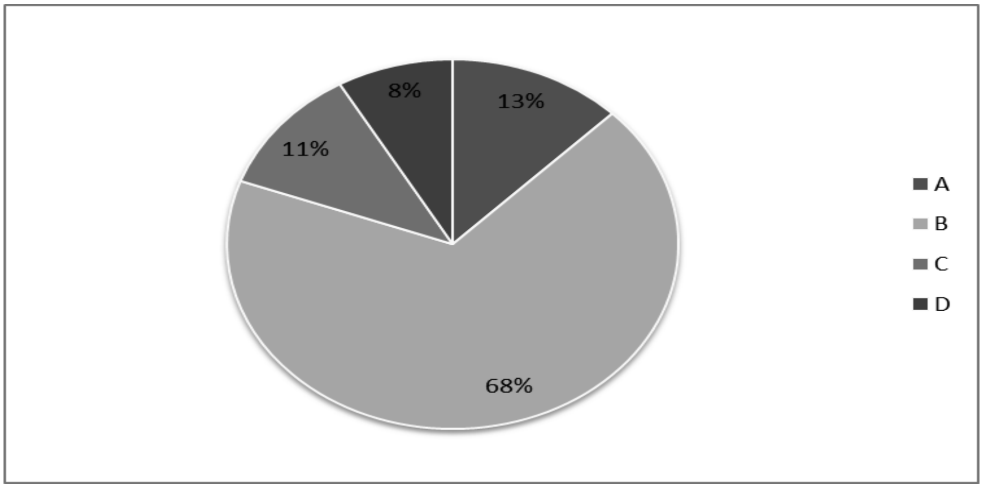

Figure 3. Control group: question 1. Percentage of students answering correctly.

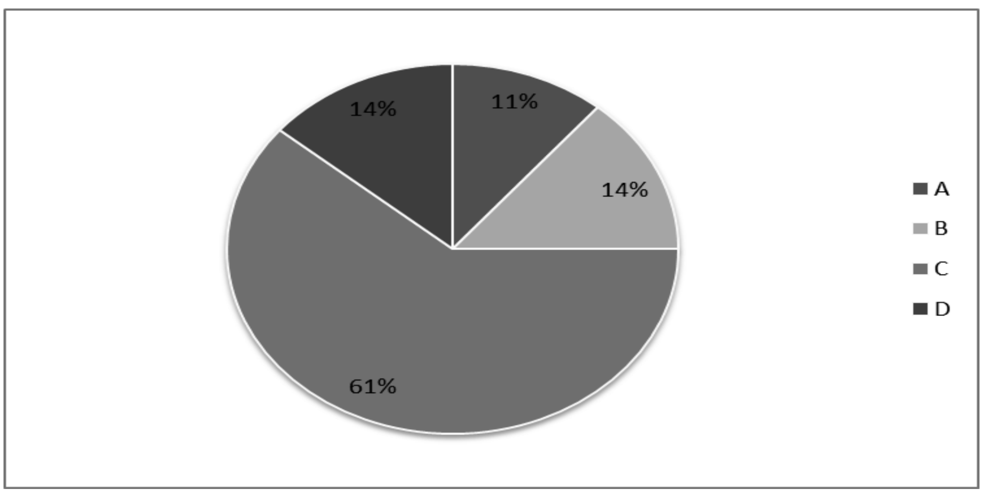

Figure 4. Control group: question 2. Percentage of students answering correctly.

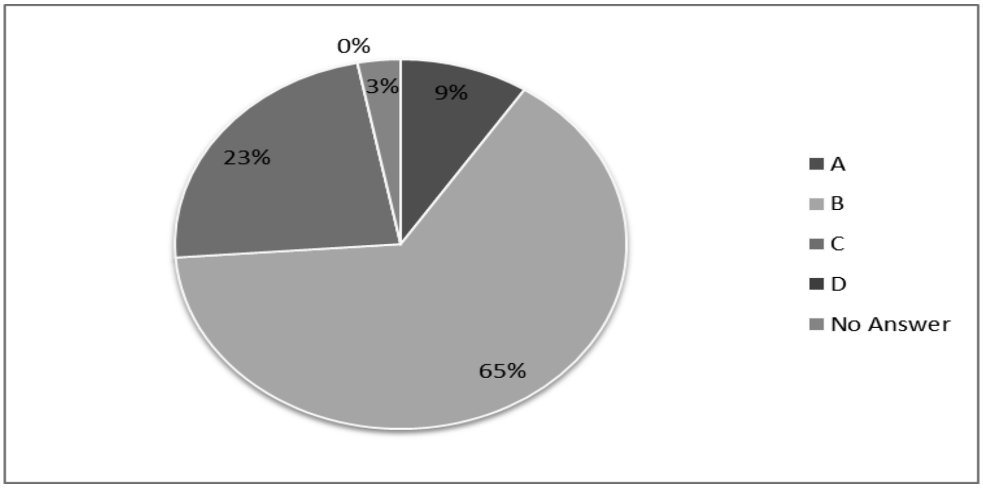

Figure 5. Experimental group: question 1. Percentage of students answering correctly.

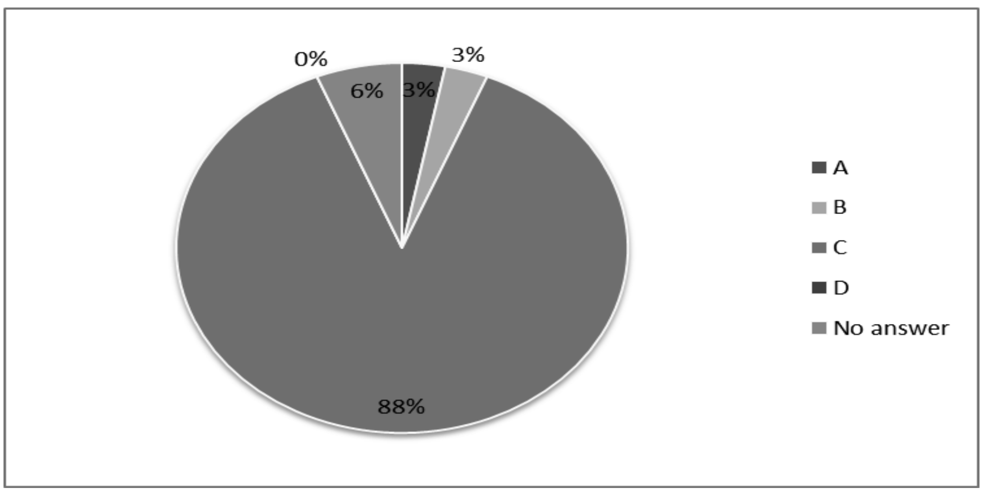

Figure 6. Experimental group: question 2. Percentage of students answering correctly.

Evaluating the Student Process

After percentages were reviewed, grounded theory and constant comparisons were utilized to determine common themes found within the students' processes. It was evident that many students worked problems in similar ways, common themes noted for question 1 were: utilizing a running equation, showing no work, working going down using a bar, and breaking the problem up. Common themes noted in question 2 were: putting numbers in for each shape, choosing a number for the pentagon. See Figure 7 and 8 to see the strategies used for these two questions.

For question 1, many students in the control group (n = 28) and experimental group (n = 33) worked the problem going down using the bar. Seventeen students in the control group utilized a running equation when solving the question whereas in the experimental group only eight students used a running equation. There were 13 students in the control group who showed no work at all for question 1, while there were only three students in the experimental group who showed no work. In the experimental group, one student created a data output chart to show their work, and 13 students in the experimental group broke each equivalent statement in its own problem.

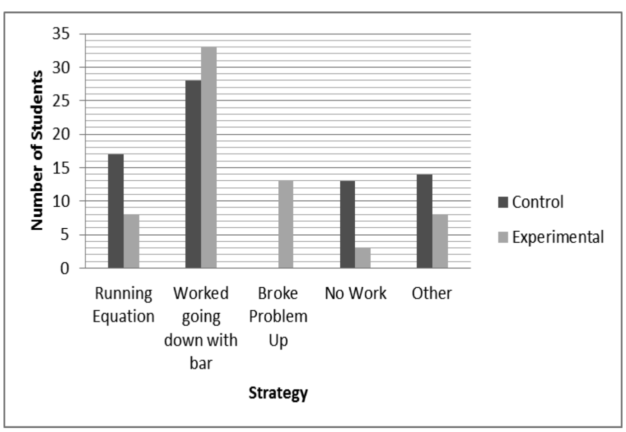

Figure 7. Question 1 strategies. These were the strategies utilized by the control and experimental group for question 1.

For question 2, fifty students in the control group did not show any work when solving this question, while in the experimental group only three students did not show their work. There were 13 students in the control group who placed numbers inside each shape to show equivalence, while the experimental group had 17 students place numbers inside each shape. For the control group, only 2 students chose their own number for the pentagon, while the experimental group had 37 students choose a number for the pentagon. There was also one student in each group that crossed out the pentagon. These strategies and results are further discussed in the subsequent section.

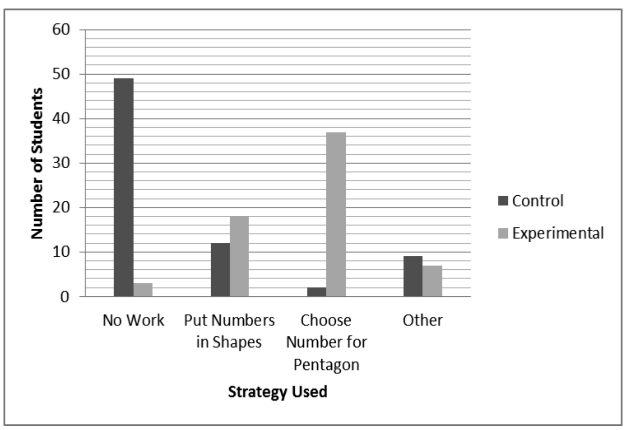

Figure 8. Question 2 strategies. These were the strategies utilized by the control and experimental group for question 2.

Discussion

Many students lack a solid understanding of the equal sign as a relationship between both sides of the equation, throughout their elementary schooling they are exposed mostly to the procedural view of the equal sign. The two equivalence questions in this study are a leading example of how students have misconceptions about the equal sign, and results support the use of atypical type statements in a classroom in order to increase understanding and deter misconceptions from occurring.

For question 1, the control group had a slightly higher percentage correct (68%) than the experimental group (65%). However, obtaining the correct answer is not always the most important, but rather how the students show their work. This is where the researcher can gain better insights into how students think. Because this study is focusing on understanding the equal sign, the experimental group had more students show their work by breaking the equivalent statements into separate parts and working the problem going down using the bar. The experimental group also had fewer students utilizing the running equation, which is very important because it is these running equations that create misconceptions for students when they enter higher-level mathematics. The results from question 1 support the theory that exposing students to atypical equivalent statements, (i.e. operators on the right hand side, operators on both sides of the equals sign, and the same number equal to itself) lead to greater understanding of the equal sign as a balance between both sides of the equation.

For question 2, the experimental group (88%) outperformed the control group (61%) significantly. Not only did the experimental group outperform the control group on the question, but also this group was able to show their work and prove their answers. Only three students showed no work from the experimental group. Many (n = 50) students in the control group answered the question without showing any work. This could lead the researcher to believe two things: either the student did not know how to start the problem because there were no numbers and only shapes, and they were not able to transfer the meaning of the equal sign to each shape; or the student could solve the problem using mental math and their understanding of the equal sign without showing their work. Regardless of the reason, students in the experimental group support the use of atypical equivalence statements because they were able to show a higher level of understanding of the equal sign by choosing their own number for the pentagon even though one was not given. Most students also ended with an equivalence statement, where the same number was equal to itself.

It is evident from the results of this study that exposing students to atypical equivalence statements lead to an increased understanding of the relational meaning equal sign, not only answering the question correctly, but also being able to accurately display work to prove their answer. Many students utilize running equations and never realize that these types of misconceptions hinder their learning in algebra type questions. For example, when students solve problems that contain operations on both sides and they are solving for a variable. Students who use these running equations automatically perform the operation on the left hand side and write the result in for the variable, without ever realizing the equal sign is a balance between the two sides, and that both operations should equal the same thing.

Furthermore, incorporating atypical type equivalence statements, as mentioned earlier in this paper, will increase student understanding of the equal sign and expose them to statements they are unfamiliar with. Exposing students to these unfamiliar statements requires students to rethink their understanding of the equal sign, and begin to view it as a balance between both sides and not as an operation, or the answer comes next.

The next step for this study would be to create curriculum or lessons teachers could utilize in their classroom to teach equality in their classroom, so their students are better prepared when they enter higher-level mathematics courses. Because the results from this study support the implementation of atypical equivalence statements early in classrooms in order to deter common misconceptions of the equal sign, and also only viewing the equal sign as an operation and not as a relationship between both sides. These concepts are crucial to success in algebra classes.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The equal sign is a very important concept, which often gets forgotten especially in early mathematics classrooms (Ball, 1990; Capraro, Ding, Matteson, Capraro, & Li, 2007; Davis & Simmt, 2006; Falkner, Levi & Carpenter, 1999; Jones & Pratt, 2005; Knuth, Stephens, McNeil, & Alibali, 2006; Li, Ding, Capraro & Capraro, 2008; Prediger, 2009; Sherman & Bisanz, 2009). Without a solid understanding of the equal sign, students are less successful in higher-level mathematics classrooms. Many students only experience the operational meaning of the equal sign, meaning the answer goes next, and never grasp the notion the equal sign is a balance between the left and right hand sides. Therefore, it is vital researchers study pre-service teachers' understanding of the equal sign, how they would correct student misconceptions and also study student misconceptions in order to offer suggestions to deter these misconceptions from occurring in the future. Both of these studies together provide pre-service teachers, current teachers, and university teacher educator programs with research supporting the use of atypical equivalence statements in classrooms, as well as requiring pre-service teachers the opportunity to evaluate student work often and discuss strategies for correcting misconceptions.

The first article examines pre-service teachers conceptions of the equal sign and their ability to evaluate student work and offer suggestions for correcting student mistakes. This study supports the notion that upon completion of a problem-solving course, pre-service teachers have a better understanding of the equal sign as compared to students in previous studies (Alibali, 1999; Prediger, 2009; Stephens, 2006) and may be ready to teach the equal sign as a relationship between numbers and not as an operation. Pre-Service teachers were required to evaluate student work in order to identify common student mistakes. Reviewing student work and being aware of these misconceptions will assist pre-service teachers in preventing these misconceptions and look for these misconception situations in their own future classrooms. This study also encouraged the use of atypical type equations in order to build students' knowledge and understanding of the equal sign.

Results from the first study led to the second study, where student understanding and work was evaluated to gain a deeper understanding of students' perceptions of the equal sign. The control group took a test and two of the test items were evaluated to check for student understanding and proof of the answer in their work. The experimental group participated in lessons in which students were required to interpret the equal sign as more than an operation, and then they were given the same two questions. Results from this study support the use of presenting atypical equivalence statements to students, which tended to increase student knowledge of the equal sign and have a positive impact on future student success as measured by the two task items.

Both of these studies support increasing understanding of the equal sign and the misconceptions students possess regarding the equal sign. University teacher education programs and public, charter, and private schools should utilize these results to require pre-service teachers and teachers to evaluate student work and teach the equal sign as a relationship between both sides of the equation and not as an operation.

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APPENDIX A

Randomized Block Design for Think Aloud Interviews

Participant	Interview Process
1	I_1 I_2 I_3
2	I_1 I_3 I_2
3	I_2 I_1 I_3
4	I_2 I_3 I_1
…	…

I_1 is the same question and interviewer throughout the interview process
I_2 is the same question and interviewer throughout the interview process
I_3 is the same question and interviewer throughout the interview process

APPENDIX B

Pre-Service Teacher Test Item

18. You are the teacher of a student who worked on a problem and got an answer. Please choose the best possible answer and then (1) explain what you think was the student's problem solving process, (2) evaluate the student's process, and (3) evaluate the student's answer.

A) The student's answer is correct

B) The student's answer is incorrect

C) The student's work shows knowledge of arithmetic

D) The work is incorrect but the answer is correct.

E) The student's work does not match the problem.

Splendiferous bought 4 apples and 3 oranges. She ate 1 apple and squeezed two oranges for juice. Later she realized she had 2 apples and 3 oranges in the refrigerator. How many pieces of fruit does she have in all?

$$4 + 3 = 7 - 1 = 6 - 2 = 4 + 2 = 6 + 3 = 9$$

APPENDIX C

Fifth Grade Equivalence Test Questions

1. Alan started a problem with a one-digit number. He multiplied the number by 3, added 8, divided by 2 and subtracted 6, and got the same number he started with. What was the number Alan started with?
    a. 2
    b. 4
    c. 6
    d. 8
2. Tracey wrote a code where shapes stand for numbers. Her code included the following statements:

    △ = 6

    ○ = 8

    Which of the following equations must be true?
    e. 4 + ⬠ = ⬠ + △ + △
    f. 5 + ⬠ = ⬠ + ○ + ○
    g. 6 + ⬠ = ⬠ + △
    h. 7 + ⬠ = ⬠ + ○

APPENDIX D

Sample Atypical Equivalence Statements Used

Are these statements correct? How do you know?

1. 15 = 15

2. 10 = 5 x 2

3. 4 + 8 = 12 – 6 = 6

4. 10 + 4 = 9 + 5

   For the next two questions use ☺ = 1

5. ☺ + 4 = ☺ + 7

6. ☼ + 1 = ☼ + ☺

Which number makes the statement true?

1. 5 + 6 = ____ + 3

2. 13 – 7 = ____ – 5

3. 12 + 9 = ____ – 17

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